Robustness of Parallel Multi-Rate A/D Converters to Anti-Aliasing Filter Non-Idealities

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Abstract— **Any digitization system must be preceded by an anti-aliasing filter. For wideband high frequency applications, parallel multi-rate conversion systems such as time-interleaved or hybrid filter bank analog-to-digital converters (resp. TI-ADC or HFB) are attractive solutions. This paper compares the robustness of both techniques with respect to non-idealities of the anti-aliasing filter (AAF). Theoretical results show that the signal-to-noise ratio (SNR) degradation due to out-of-band signals is lesser for HFBs than for TI-ADCs, provided that the analysis filters of the HFB are selective enough. Simulation results show that this is the case even for low-order analysis filters in the case of a four-channel HFB.**

I. INTRODUCTION

In the context of a cognitive radio application, the radio receiver should deal with wideband and high-frequency signals. Concerning the ADC, parallel multi-rate ADCs such as time-interleaved or hybrid filter bank analog-to-digital converters (resp. TI-ADC or HFB) are attractive because such structures make it possible to enlarge the bandwidth by reusing already existing designed ADCs. Even if TI-ADCs have been developed since the 80's, they are still widely studied [1], [2]. The reason is certainly their ease of implementation. On the other hand, HFBs, which were introduced in the 90's, are more complex than TI-ADCs because they include an analog bandpass filter bank, called analysis bank. Even if the realization of the analog filter bank is tricky, HFBs are still studied [3], [4]. Compared to TI-ADCs, HFBs partially solve the problem related to peak to average power ratio (PAPR). Also, as seen in this article, another advantage of HFBs is that the analysis bank contributes to the attenuation of out-of-band signals. Therefore, the preceding anti-aliasing filter (AAF) constraints can be relaxed in the case of an HFB compared to a TI-ADC. As far as the authors know, this aspect has not been yet explicitly highlighted and quantified in publications.

This paper presents a theoretical approach to roughly evaluate the impact of AAF non-idealities on the conversion accuracy. For this theoretical study, very simple models are considered for all filter magnitudes. Then, simulations take into account more realistic filters based on elliptic and Butterworth topologies. The results make it possible to compare the impact of AAF non-idealities on several structures: a unique ADC (which may be considered as a reference), a Time-Interleaved (TI-) ADC and a Hybrid Filter Bank (HFB).

II. THEORETICAL APPROACH

This study is conducted at system level with some approximations in the shapes of the filters. Assuming a band-pass conversion, let us consider the conversion of the band $[nB,(n+1)B]$ at sampling frequency $2B$. If the AAF does not perfectly eliminate the out-of-band signals, aliasing will occur. Fig. 1 shows the model considered for the AAF magnitude. First, it is assumed that the AAF magnitude is perfect in the useful band (no ripple). Second, it is assumed that the signal is not perfectly eliminated on both sides of the band. Nevertheless, beyond B on both sides, it is assumed that the signals can be neglected. This hypothesis is reasonable because the antenna and the LNA perform a preliminary band-pass filtering of the received signal. Starting from this model, it is possible to quantify the impact of AAF non-idealities on the conversion quality for the studied structures.

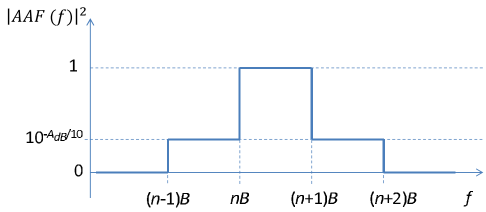

Fig. 1. Anti-aliasing filter magnitude modelling

As shown in Fig. 1, A_{dB} is the out-of-band attenuation. It is assumed that the input signal has a uniform power spectral density. Denoting σ_x^2 the variance of the useful signal in the band $[nB,(n+1)B]$, and σ_A^2 the variance of the signal in each band $[(n-1)B,nB]$ and $[(n+1)B,(n+2)B]$, one has: $\sigma_A^2 = \sigma_x^2 10^{-A_{\mathrm{dB}}/10}$.

In the case of a single ADC sampling at $2B$ rate, there are two aliasing terms. Thus, the resulting signal-to-noise ratio (SNR, in dB) is

$$\mathrm{SNR}^{(\mathrm{ADC})} = 10\log\left[\frac{\sigma_x^2}{2\sigma_A^2}\right] = A_{\mathrm{dB}} - 10\log(2). \quad (1)$$

For an M-channel TI-ADC, let us consider the simplest model, exposed in Fig. 2, where $T = 1/2B$. The output $y(n)$ is obtained by interleaving the x_k's. So, the resulting SNR is the same as for a unique ADC:

$$\mathrm{SNR}^{(\mathrm{TI-ADC})} = 10\log\left[\frac{\sigma_x^2}{2\sigma_A^2}\right] = A_{\mathrm{dB}} - 10\log(2). \quad (2)$$

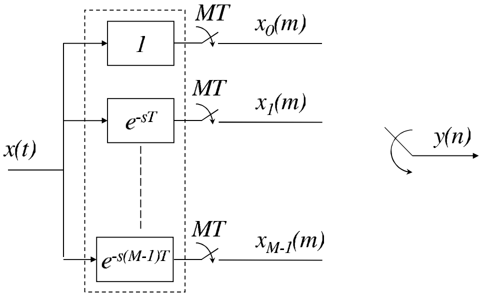

Fig. 2. TI-ADC model

Concerning the HFB, let us consider the classical model of Fig. 3. In order to go ahead in the calculation, it is necessary to introduce the equivalent polyphase representation of the digital bank given in Fig. 4, as performed in [5].

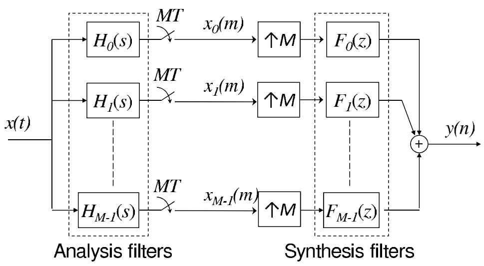

Fig. 3. HFB model

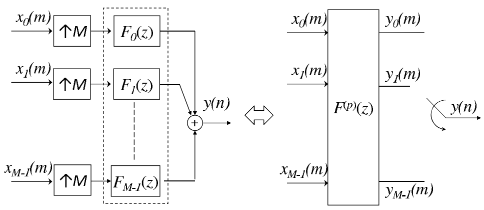

Fig. 4. HFB synthesis filter bank and its polyphase representation

Assuming the independence of the errors in the signals $y_i(n)$, the variance of the total error at the output $e(n) = y(n) - x(nT)$ can be expressed as the average variance

$$\sigma_e^2 = \frac{1}{M}\sum_{i=0}^{M-1}\sigma_{ey_i}^2, \quad (3)$$

where $\sigma_{ey_i}^2$ is the variance of the error in $y_i(n)$. Using the equivalence between the digital bank and its polyphase representation, equation (3) above is developed as

$$\sigma_e^2 = \frac{1}{M}\sum_{i=0}^{M-1}\sigma_{ex_k}^2\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|F_k(e^{j\omega})\right|^2 d\omega, \quad (4)$$

where $\sigma_{ex_k}^2$ is the variance of the error in x_k and $F_k(e^{j\omega})$ is the frequency response of the FIR digital filter (called synthesis filter) on path m. It is then necessary to define a model for the magnitude response of the synthesis filters. To this end, a simple model must first be chosen for the analysis filters. Fig. 5 shows the chosen model for the analysis filter of channel m. Each analysis filter has a magnitude of 1 in its dedicated subband and an attenuation of a_{dB} for other frequencies.

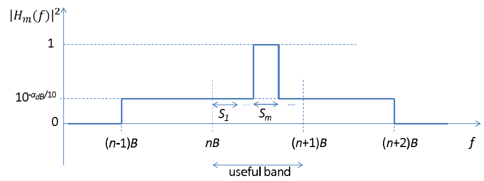

Fig. 5. Magnitude response of the analysis filter of channel m

The corresponding optimal synthesis filters can be obtained by minimizing a least mean squares criterion, such as the LMSGA criterion [6]. Such a method provides a synthesis bank that jointly minimizes the reconstruction error and the amplification of the quantization noise. The corresponding synthesis filter bank is given in Fig. 6.

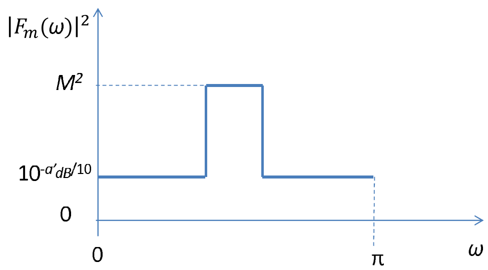

Fig. 6. Magnitude response of the synthesis filter of channel m, corresponding to the analysis filter bank given in Fig. 5

Each synthesis filter has a magnitude of M in its dedicated subband and an attenuation of a'_{dB} for other frequencies. The resulting SNR (in dB) is

$$\mathrm{SNR}^{(\mathrm{HFB})} = A_{\mathrm{dB}} - 10\log(2M) + a_{\mathrm{dB}} - 10\log\left(1 + (M-1)\frac{a'}{M^2}\right). \quad (5)$$

If a synthesis criterion limiting the amplification of the quantization noise is used, it is reasonable to assume that each synthesis filter attenuates the signal out of its dedicated band so that $a'/M << 1$. Furthermore, because the goal is to get an SNR of at least 50 dB, it is possible to neglect the last term of expression (5). Thus:

$$\mathrm{SNR}^{(\mathrm{HFB})} \approx A_{\mathrm{dB}} - 10\log(2M) + a_{\mathrm{dB}}. \quad (6)$$

Table I summarizes the above results with the comparison of the SNR due to aliasing of out-of-band signals for the three types of ADCs considered.

It can be noticed that, for an HFB, the SNR degradation is proportional to the number of channels whereas it is not the

TABLE I

SIGNAL-TO-NOISE RATIO FOR A SINGLE ADC, A TI-ADC AND AN HFB

	$\forall M$	$M=2$	$M=4$
single ADC	$A_{\mathrm{dB}}-10\log 2$	$A_{\mathrm{dB}}-3$	$A_{\mathrm{dB}}-3$
TI-ADC	$A_{\mathrm{dB}}-10\log 2$	$A_{\mathrm{dB}}-3$	$A_{\mathrm{dB}}-3$
HFB	$A_{\mathrm{dB}}-10\log(2M)+a_{\mathrm{dB}}$	$A_{\mathrm{dB}}-6+a_{\mathrm{dB}}$	$A_{\mathrm{dB}}-9+a_{\mathrm{dB}}$

case for a TI-ADC. The AAF seems therefore more critical for HFBs than for TI-ADCs. However, this degradation of $10\log(M)$ dB can easily be compensated by the attenuation of the analysis filters (i.e. a_{dB}). Thus, for a two-channel HFB, an out-of-band attenuation of 3 dB is sufficient to get the same degradation as for a TI-ADC. For a four-channel HFB, an attenuation of 6 dB is sufficient to recover the same degradation as for a TI-ADC. Furthermore, if a larger attenuation is even taken, it means that in order to get the same performance as for a single ADC or a TI-ADC, it is possible to relax the attenuation constraint on the AAF (A_{dB}).

III. SIMULATION RESULTS

In this part of the study, we estimate with system-level (Matlab) simulations the impact of the AAF on the performance of the converter with more realistic models of frequency responses for the AAF, and for both the analysis and synthesis banks in the HFB case. Still, as in the theoretical study, we consider that outside the $[(n-1)B,(n+2)B]$ band, the signals can be neglected. In practice, this assumption can be justified by the fact that the antenna+LNA reception chain has a globally lowpass behavior, which filters out the input signal for frequencies far from the converted band in the chosen scenario.

A. AAF noise power formulas

To compute the performance of the converter, we first need to estimate the total power W_{AAF} of the noise on the output signal caused by the non-ideality of the AAF. This is done by simply integrating the power spectral density (PSD) of the AAF noise:

$$W_{\mathrm{AAF}} = \int_{-\frac{1}{2}}^{\frac{1}{2}} w_{\mathrm{AAF}}(f)\,\mathrm{d}f \approx \frac{1}{N_f}\sum_{f} w_{\mathrm{AAF}}(f), \tag{7}$$

the difficulty being the computation of the PSD.

In an M-path HFB, each B-wide band adjacent to the converted band produces exactly M aliasing terms on each in-band frequency (see Fig. 7 for an example with $n=1$). More precisely, any frequency component of the in-band signal on each path is aliased with M terms coming from the $[(n-1)B,nB]$ band and M others from the $[(n+1)B,(n+2)B]$ band. The frequency components of the resulting noise for each aliasing term can be expressed as

$$R_{i,m}(f) = \frac{1}{M}X(f_i)G(f_i)H_m(f_i)F_m(f), \tag{8}$$

where $1\le m\le M$ is the path index, $1\le i\le 2M$ the aliasing index, X the input signal Fourier transform, G the frequency response of the AAF, H_m (resp. F_m) the frequency response of the analysis (resp. synthesis) filter for this path, and f_i the frequency that alias with f. The $1/M$ factor is due to the fact that the power of the signal is reduced when undersampled.

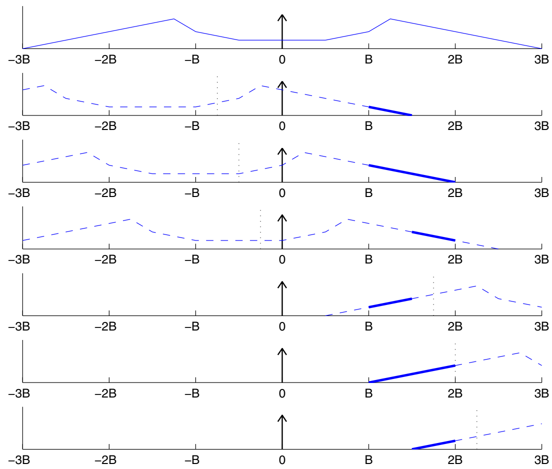

Fig. 7. Example of aliasing caused by AAF non-idealities in a four-channel HFB. The top curve represents the spectrum of the signal before undersampling. The aliasing terms produced by the $[2B,3B]$ band on the $[B,2B]$ band are represented in thick line in the other plots.

The resulting PSD of the AAF noise on the output signal is the total sum of the aliasing terms squared, multiplied by 2 to take into account the negative part of the spectrum (all signals are real-valued):

$$w_{\mathrm{AAF}}(f) = 2\sum_{m=1}^{M}\sum_{i=1}^{2M}|R_{i,m}(f)|^2. \tag{9}$$

In the TI-ADC case as in the single ADC case, each B-wide band adjacent to the converted band produces only one aliasing term on each in-band frequency. The resulting PSD of the AAF noise on the output signal is then

$$\begin{aligned} w_{\mathrm{AAF}}(f) = 2\big(&|X(2nB-f)G(2nB-f)|^2 \\ &+|X(2(n+1)B-f)G(2(n+1)B-f)|^2\big). \end{aligned} \tag{10}$$

B. Overall SNR formulas

To compute a meaningful performance of the converter in terms of SNR, other sources of noise have to be considered. In the HFB case, the main other sources of noise are the aliasing caused by the undersampling (but not due to AAF non-idealities) and the quantization. Assuming that the total power of the input signal is 1, we denote the total power of the useful signal at the output by W_{signal}, which slightly differs from 1 due to the fact that the transfer function of the HFB is not perfectly flat over the band of interest [6]. The overall SNR (in dB) is then defined as

$$\mathrm{SNR}_{\mathrm{AAF}}^{(\mathrm{HFB})} = 10\log\left(\frac{W_{\text{signal}}}{W_{\text{alias}} + W_{\text{quant}} + W_{\mathrm{AAF}}}\right), \qquad (11)$$

where W_{alias} is the total power of total aliasing, whose PSD is defined as in [6], and W_{quant} is the power of the quantization noise, which can be computed by

$$W_{\text{quant}} = 10^{\left(-\frac{6N_{\text{bits}}}{10}\right)}, \qquad (12)$$

with N_{bits} the bit resolution of the elementary ADCs.

In the TI-ADC case, the main source of noise is the quantization noise. Assuming that the elementary ADCs have perfect uniform frequency responses, the useful signal at the output has a total power of 1. The overall SNR (in dB) is then defined as

$$\mathrm{SNR}_{\mathrm{AAF}}^{(\mathrm{TI-ADC})} = 10\log\left(\frac{1}{W_{\text{quant}} + W_{\mathrm{AAF}}}\right). \qquad (13)$$

C. Numerical results

To obtain numerical values, we now need to choose a scenario on which to apply the above formulas. The scenario considered is the conversion of the band $[B, 2B]$ with $B = 420\,\mathrm{MHz}$, the band of interest $[470, 790]\,\mathrm{MHz}$ corresponding to a potentially interesting UHF band for white spaces [7]. For the HFB synthesis, a similar criterion as in [8] is used to prevent the amplification of quantization noise in the guard bands. The elementary ADCs have a resolution of 12 bits. The AAF is chosen to be an elliptic filter, with a ripple of 2 dB in its passband specified to be the band of interest and an attenuation of 65 dB outside the converted band. The minimal order for this type of filter is 14.

TABLE II

COMPARISON OF AAF NOISE AND OVERALL SNR FOR A TI-ADC AND AN HFB WITH BUTTERWORTH (BW.) ANALYSIS FILTERS

	W_{AAF} (dB)	$\mathrm{SNR}_{\mathrm{AAF}}$ (dB)
TI-ADC	−63	62
HFB (Bw. order 2)	−71	66
HFB (Bw. order 6)	−90	71

Table II shows the total power of AAF noise W_{AAF} (in dB) in the band of interest, and the overall SNR for a TI-ADC and for a four-channel HFB with Butterworth filters of a specified order as analysis filters. As expected, the higher the order of the analysis filters is, the more selective they are, and consequently the more they reduce the AAF noise. But even for the simplest type of filter (second-order), the reduction is enough to make the HFB perform better than a TI-ADC in terms of overall SNR.

On the other hand, in the sixth-order case, the quality of reconstruction is very good and the AAF noise is sufficiently attenuated so that the performance of the HFB is limited mainly by the quantization noise (12 bits correspond to an SNR of 72 dB).

The attenuation of the AAF filter can be specified so as to get a targeted performance. If a targeted performance of 66 dB is chosen for instance (which corresponds to an effective bit resolution of 11 bits), the minimal order of the necessary AAF filter is 16 in the TI-ADC case, 14 in the HFB with second-order filters, and 10 in the HFB case with 6th-order filters.

IV. CONCLUSION

This work compares HFBs and TI-ADCs regarding the in-band noise caused by the non-idealities of the anti-aliasing filter placed in front. The resulting SNR is independent of the number of channels for TI-ADCs and equivalent to the SNR obtained with a single high-speed ADC. For HFBs, a degradation of the SNR proportional to the number of channels occurs. However, this degradation may be compensated by the attenuation of the analysis filters, since the analysis filters also contribute to the filtering out of the out-of-band signals.

Simulation results are given for four-channel HFB and TI-ADC architectures. They show that, considering a given targeted SNR, anti-aliasing constraints upon the out-of-band attenuation can be relaxed in the HFB case compared to the TI-ADC case, even for low-order analysis filters. If higher orders are chosen for the analysis filters, the anti-aliasing filter constraints can be relaxed even more.

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